I can demonstrate understanding of how to solve polynomial equations.

LEVEL 1

Find all zeros of the following functions.

1. $f(x) = 2x^3 + 3x^2 - 8x + 3$; given x = -3 is a zero

$$-3 \; | \begin{array}{cccc} 2 & 3 & -8 & 3 \\ & -6 & 9 & -3 \\ \hline 2 & -3 & 1 & 0 \end{array}$$

$2x^2 - 3x + 1$

$2x^2 - 3x + 1$

$(2x - 1)(x - 1)$

$2x - 1 = 0$ 　　 $x - 1 = 0$

$2x = 1$ 　　 $x = 1$

$\boxed{x = \frac{1}{2}}$

$\boxed{\begin{array}{l} x = -3 \\ x = \frac{1}{2} \\ x = 1 \end{array}}$

2. $f(x) = x^3 - 7x + 6$; given $(x - 2)$ is a factor

$x - 2 = 0$ 　 $\boxed{x = 2}$

$$2 \; | \begin{array}{cccc} 1 & 0 & -7 & 6 \\ & 2 & 4 & -6 \\ \hline 1 & 2 & -3 & 0 \end{array}$$

$x^2 + 2x - 3$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{16}}{2}$

$x = \dfrac{-2 \pm 4}{2}$

$x = \dfrac{-2 + 4}{2} = 1$

$x = \dfrac{-2 - 4}{2} = -3$

$x = 2, 1, -3$

3. Is $(x - 3)$ a factor of $f(x) = 5x^3 + 2x^2 - 9x + 5$? Explain.

$x - 3 = 0$

$x = 3$

$$3 \; | \begin{array}{cccc} 5 & 2 & -9 & 5 \\ & 15 & 54 & 135 \\ \hline 5 & 17 & 45 & \boxed{140} \end{array}$$

No because there is a Remainder

LEVEL 2/3

4. Find all (real and complex) roots of the polynomial $f(x) = x^3 - 3x^2 + 5x - 15$ given x = 3 is a solution.

$$3 \; | \begin{array}{cccc} 1 & -3 & 5 & -15 \\ & 3 & 0 & 15 \\ \hline 1 & 0 & 5 & 0 \end{array}$$

$x^2 + 5 = 0$

$x = \pm \sqrt{5}$

$x = 3$

5. Find all (real and complex) roots of the polynomial $f(x) = x^4 - 9x^3 + 23x^2 - 81x + 126$ (You will need to divide twice)

$$x = 2 \; | \; 1 \quad -9 \quad +23 \quad -81 \quad +126$$
$$\underline{\quad\quad 0 \quad 2 \quad -14 \quad 18 \quad 126}$$
$$2 \; | \; 1 \quad -7 \quad 9 \quad -63$$

$$x^3 - 7x^2 + 9x - 63$$

$$x = 7 \; | \; 1 \quad -7 \quad 9 \quad -63$$
$$\underline{\quad\quad\quad 7 \quad 0 \quad 63}$$
$$7 \; | \; 1 \quad 0 \quad 9 \quad 6$$

$$x^2 + 9$$

$$\boxed{x = 2, 7 \\ 3i, -3i}$$

$$x = \frac{0 \pm \sqrt{0^2 - 4(1)(9)}}{2}$$
$$x = \frac{0 \pm \sqrt{-36}}{2}$$
$$x = \frac{0 \pm 6i}{2}$$
$$x = \pm 3i$$

6. Write and solve a polynomial equation to find the dimensions of the solid with the given volume:

a. $V = 42$ cm³

x - 2

x

2x + 1
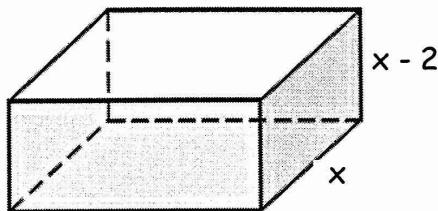
$$(2x+1)(x)(x-2) = 42$$
$$x = 3.5$$

Dimensions: 8 × 3.5 × 1.5

b. $V = 82$ m³

x - 2

x - 3

x - 5
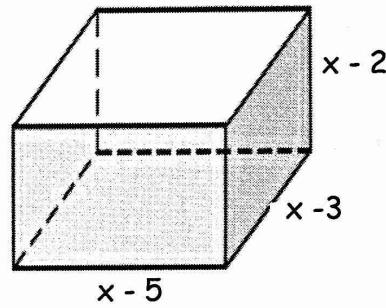
$$(x-5)(x-3)(x-2)$$
$$x = 7.9$$
$$(2.9)(4.9)(5.9)$$

Dimensions: (2.9)(4.9)(5.9)

7. You have a piece of paper which you will use to make a box by cutting x inches from each corner as shown then folding up the sides.

$$0 < x < 6$$

a. Find all possible dimensions if the Volume is 150 in³

$$x = 1.27$$
$$x = 3.293$$

x   x
x   x

12 in

x   x
x   x

15 in
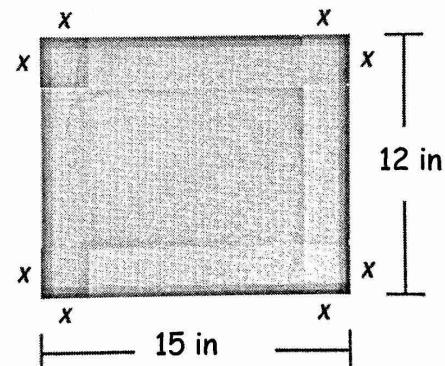
b. What is the maximum volume for the given solid?

Max volume 177.23

$x = 2.2$ that will give you max volume